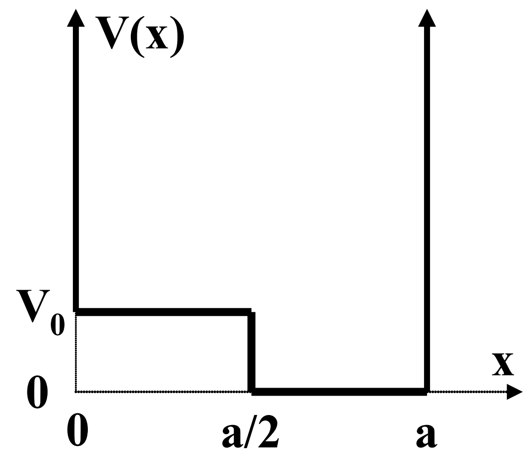
1. A particle of mass m is trapped in a one-dimensional asymmetrical infinite square well as shown in the figure. [5,5,5,5,5]

a) What condition must be satisfied to treat this problem using perturbation theory?
b) Write down the un-perturbed Hamiltonian, eigenenergies and eigenfunctions.
c) What is the perturbing Hamiltonian?
d) Use first order perturbation theory to calculate all of the eigenenergies of the particle.
e) What are the first-order corrected eigenfunctions?



2. Evaluate the following commutators by breaking them down to elementary commutators involving only single powers of operators (e.g. $[a, b]$, rather than $[a^2, b]$, etc.). In other words use only $[x, p_x] = [y, p_y] = [z, p_z] = i\hbar$, $[x, y] = [x, p_y] = 0$, etc.

[7,6,6,6]

a) $[x, L_x]$
b) $[z, L_y]$
c) $[y, L_+]$
d) $[L_z, r^2]$

3. The wave function of a rigid rotator with a Hamiltonian H = $\mathbf{L}^2/2I$, where I is the (constant) moment of inertia, is given by [8,8,9]

$$\psi(\theta,\phi)=\sqrt{\frac{3}{4\pi}}\,\sin\theta\,\sin\phi$$

a) What values of L_z will be obtained if a measurement is carried out and with what probability will these values occur?

b) What is $\langle L_x \rangle$ for this state?

c) What are the eigen-energies of the Hamiltonian?

4. A hydrogen atom is prepared in the $3d_{5/2}$ state with $m_j = +3/2$. (Recall the notation: nℓ$_j$, where "ℓ" is a code letter.) The total wavefunction (including spin) in this state is denoted ψ. [5,5,5,10]

a) On a single graph make a careful sketch of the various components of the effective potential $V_{eff}(r)$ for the atom in this state. Also draw $V_{eff}(r)$.

d) Calculate the total energy for the atom in this state (to zeroth order), and on the same diagram note this energy. (no need to find the exact numerical value)

e) Find and sketch the <u>probability</u> $P(r)$ of finding an electron in this state between r and $r + dr$, as a function of r (be careful to get the units right so that $P(r)dr$ is dimensionless). Write down the precise functional dependence of the probability on r for $r \to 0$ and $r \to \infty$.

f) Calculate the expectation values of the following observables:

i) H (Hamiltonian)

ii) L^2

iii) J^2

iv) S_z

v) L_z